

ORIGINAL REPORT

BILATERAL ROBOT THERAPY BASED ON HAPTICS AND REINFORCEMENT LEARNING: FEASIBILITY STUDY OF A NEW CONCEPT FOR TREATMENT OF PATIENTS AFTER STROKE

Valentina Squeri, Eng^{1*}, Maura Casadio, PhD^{1*}, Elena Vergaro, Eng^{2*}, Psiche Giannoni, PT³, Pietro Morasso, Eng^{1,2} and Vittorio Sanguineti, PhD^{1,2}

From the ¹Italian Institute of Technology, ²NeuroLab, DIST, University of Genova and ³ART Education and Rehabilitation Center, Genova, Italy. *Valentina Squeri, Maura Casadio and Elena Vergaro contributed equally to this study.

Objective: To carry out a preliminary feasibility study of a new concept of robot therapy for severely impaired patients after stroke.

Design: A haptic manipulandum connected to a bar that can rotate freely while providing a measure of the rotation angle. The controller combines a bilateral reaching task with the task of balancing the action of the 2 arms. Reinforcement is given to the subject in 2 forms: audio-visual and haptic by means of adaptable force fields.

Patients: Four highly paretic patients with chronic stroke (Fugl-Meyer score less than 15).

Methods: The training cycle consisted of 5 sessions over a period of 2 weeks. Each session (45 min) was divided in blocks of 10 pairs of forward/backward movements. Performance was determined by evaluating the number of successful movements per session, the session-by-session decrease in the assistive field, the mean reaching time, and the mean stopping field.

Results: All subjects could understand the task, appreciated it and improved their performance during training. The reaching movements became smoother and quicker; balance errors and the magnitude of the resisting field were consistently reduced.

Conclusion: Bilateral robot therapy is a promising technique, provided that it self-adapts to the patient's performance. Formal clinical trials should address this point.

Key words: rehabilitation, robotics, stroke, touch perception, reinforcement, learning.

J Rehabil Med 2009; 41: 961–965

Correspondence address: Valentina Squeri, Istituto Italiano di Tecnologia, Via Morego 30, IT-16163 Genova, Italy. E-mail: valentina.squeri@iit.it

Submitted March 16, 2009; accepted May 19, 2009

INTRODUCTION

Over the past years evidence has mounted regarding the capacity of the central nervous system (CNS) to alter its structure and function throughout all sorts of life experiences, including injuries to the CNS, in a complex network of interacting processes (1–4). Animal models of focal brain injuries suggest that behaviour is probably the most powerful modulator of post-injury recovery (5,

6); thus, beyond the initial critical period of self-repair (7), the principal process responsible for functional recovery is the use-dependent reorganization of neural mechanisms made possible by neural plasticity (8). Moreover, imaging data suggest that circuitry in motor cortices on both sides of the brain is modified during recovery (9), and this has led to the concept that bilateral movement permits inter-hemispheric facilitation of the limbs (10).

This is the main motivation for the design of robotic or mechatronic devices that aim at bilateral training of the normal and the paretic arm. Early prototypes of bilateral trainers were developed at the VA Palo Alto Center (11), based mainly on the so-called mirror image movement enabler concept (MIME) in which a robot manipulator applied forces to the paretic arm during goal-directed movements, keeping it in mirror-symmetry with the unaffected arm whose position was monitored by a position digitizer. Simple, low-cost bilateral arm trainers have also been developed and tested. Bilateral Arm Training, Auditory-Cued (BATRAC) is an example of such systems: it is a one degree of freedom custom-made mechanical arm trainer (12) that allows auditory cued patients to move two unyoked T-grips forward and backward in a parallel or alternate fashion. Another system in the same category is Reha-Slide (13), which allows unilateral or bilateral training of up to 3 degrees of freedom of the shoulder, elbow and wrist.

These bilateral trainers are aimed in particular at severely impaired patients who cannot carry out full extension reaching movements with the paretic limb without suitable assistance and thus are not eligible for conventional treatment approaches, including the promising constraint-induced movement therapy (14). However, in the previously mentioned bilateral arm trainers, movements of the paretic arm are activated in a passive way, using the unaffected arm as the “primus movens” in order to overcome the inability of the paretic limb to carry out the prescribed movements.

In this paper, we propose an alternative concept: to use the robot as “primus movens” and combine the bilateral reaching task with the task of balancing the action of the 2 arms, according to a reinforcement learning paradigm. In this way the relationship between the 2 limbs is not of the master-slave type and the patient is strongly motivated to balance and co-ordinate the activation of the 2 limbs. This new bilateral training concept was implemented by means of a simple mechanical extension

of the haptic robot Braccio di Ferro (BdF) (15) and an original haptic interaction scheme. The mechanical extension consists of a bar connected to the end-effector of the robot. The bar can rotate freely and the corresponding rotation angle is measured by a coaxial rotation sensor. The subject holds 2 handles at the 2 ends of the bar and is required to balance the forces applied by the 2 hands in such a way to reach a target and, at the same time, maintain the bar at a prescribed angle. The reinforcement learning scheme is expressed by means of suitable force fields that adapt to the patient's performance. The feasibility of this training concept was tested with a preliminary clinical study that yielded promising results with 4 severely impaired patients. The approach can be adapted easily to any haptic robot that, as BdF or MIT-Manus (16), allows bi-directional human-robot interaction and the fine control of the interaction forces.

METHODS

Experimental apparatus

The robot, BdF, is a planar manipulandum with 2 degrees of freedom, designed at the University of Genoa (15). Its most relevant features are: (i) large planar workspace (80 × 40 cm ellipse); (ii) rigid mechanical structure with direct drive of 2 brushless motors, designed in order to have low intrinsic mechanical impedance at the end-effector; (iii) large available force at the handle (continuous force > 50 N; peak force > 200 N); (iv) impedance control scheme that allows a bi-directional, smooth haptic interaction between the robot and the patient. Low mechanical impedance means that when the robot controller is off the subject perceives a virtually weightless, frictionless, and noiseless manipulandum. This also significantly improves the safety of the robot.

For the purpose of this study, the handle of the manipulandum, which is typically grabbed by the paretic hand of the patient, was substituted by a horizontal bar (Fig. 1) hinged in the middle and connected to the terminal part of the robot. This was facilitated by the modular design of BdF that allows easy modification of the geometry of the arm, the

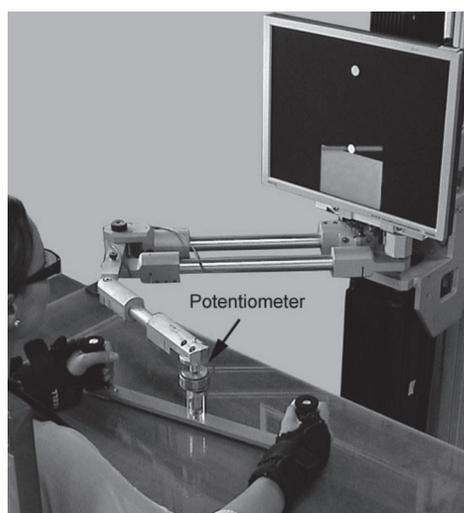


Fig. 1. The haptic robot Braccio di Ferro, modified by mounting a horizontal bar for bimanual co-ordination. The bar is free to rotate around a vertical hinge. The rotation angle is measured by a potentiometer. The computer screen displays the target and the position/orientation of the bar. The task is to reach the target with an approximately horizontal bar ($\pm 4^\circ$). Note the wrist holders, used in skate-boarding.

operational plane and assembly/disassembly of additional mechanical parts, tailored for specific experimental protocols. As shown in Fig. 1, the patient grabs 2 handles, symmetrically positioned with respect to the central hinge. The distance between the handles can be adjusted in order to match the distance between the shoulders of the patient. The rotation of the bar is not actuated, but the rotation angle is measured by a potentiometer.

Subjects were seated on a rigid chair with the shoulders strapped to it in such a way to prevent forward displacement of the trunk. Moreover, both wrists were prevented from flexing/extending, by means of comfortable holders, as used in skate-boarding.

A light support was connected to the forearms in order to allow low-friction sliding on the horizontal surface of a wooden table covered with a plexiglass support. Movements were restricted to the horizontal plane, in order to avoid the influence of gravity. The position of the seat was also adjusted in such a way that, with the cursor pointing at the centre of the workspace, the elbow and the shoulder joints were flexed approximately 90° and 45° , respectively. A 21" liquid crystal display (LCD) computer screen was placed in front of the subjects, approximately 1 m away, at eye level.

Subjects

Four subjects with chronic stroke (2 males, 2 females) volunteered to participate in this study (Table I). They were recruited from among outpatients of the ART Education and Rehabilitation Center, Genoa. Inclusion criteria were: (i) diagnosis of a single, unilateral stroke verified by brain imaging; (ii) sufficient cognitive and language abilities to understand and follow instructions; (iii) chronic (at least one year after stroke) and stabilized conditions (at least one month before entering robot therapy); and (iv) high impairment level (Fugl-Meyer score, arm section (FMA) score less than 15 (range 0–66)). Four control subjects tested the system, providing reference performance levels.

The research conforms to the ethical standards on human experimentation and with the 1975 Declaration of Helsinki, as revised in 1983. Each subject signed a consent form that conforms to these guidelines. The robot training sessions were carried out at the Neurolab of the Department of Informatics, Systems and Telematics of the University of Genoa, under the supervision of a physiotherapist with more than 20 years of experience.

Experimental protocol and robot assistance

The subjects sat in front of a computer screen that displayed the target (a circle of 2 cm diameter) and a bar, positioned according to the robot end-effector co-ordinates and oriented according to the potentiometer reading: the centre of the bar was marked by another circle with the same diameter and different colour.

The target switched between 2 positions separated by 20 cm in the anterior-posterior direction with respect to the body of the subject. The task consisted of reaching the target with the centre of the bar, while maintaining the bar perpendicular to the nominal movement direction. A range of $\pm 4^\circ$ was chosen for the tolerated orientation error, after testing the system with the control subjects.

A visual (colour) code and an acoustic feedback were used in order to reinforce correct performance. The colour of the bar changed depending on its orientation: it was green if the angular error was kept

Table I. Clinical data of the subjects

Subject	Age, years	Sex	DD, years	Aetiology	PH	Ash	FMA
S1	74	M	4	I	L	3	4
S2	48	F	4	H	L	2	13
S3	32	F	3	I	L	2	9
S4	62	M	1	I	L	1+	11

Ash: Ashworth score (0–4); DD: disease duration; F: female; FMA: Fugl-Meyer score, arm section (0–66); H: haemorrhagic; I: ischaemic; L: left; M: male; PH: paretic hand; R: right.

inside the prescribed range and it became red when the error became larger. Moreover, an unpleasant sound signalled that the orientation error was outside the threshold and a pleasant sound marked that the target was reached.

As soon as a subject reached a target, that target was switched off and the other target was activated, thus inducing a sequence of forward/backward movements that became quicker and quicker as performance improved.

Motor performance was also reinforced by the haptic interaction between the robot and the patient (Fig. 2). Such interaction was implemented by a virtual haptic environment (Appendix I) that was obtained by combining different force fields:

- *Assistive field.* This force field is applied to the manipulandum and is directed to the current target. It is activated in a smooth way, when a target is presented, and it stays on throughout the whole movement until the target is reached. The magnitude of the field is personalized for each patient and is selected according to a *minimally assistive strategy* (17). This means that an initial test session was used for allowing each patient to become familiar with the system and for evaluating the minimum amplitude of the force field that is capable of inducing the movement initiation of the paretic limb: for the 4 patients this force amplitude ranged between 8 and 25 N. The field magnitude was reduced in following sessions as performance improved. In this way the unaffected arm was freed from the task of providing the basic action that allowed the paretic arm to approach the target, and a master-slave situation between the 2 limbs was avoided. At the same time, the strategy avoided the establishment of a master-slave relationship between the robot and the paretic arm, thus fostering the emergence of voluntary control patterns. In a sense, the assistive field was a positive reinforcement to the motor control circuitry of the paretic limb.
- *Stopping field.* This is a strong elastic field (with a stiffness of 1200 N/m), which opposes the movement and is activated when the bar orientation error exceeds the threshold of $\pm 4^\circ$; it is switched off as soon as balance is recovered. The transition from activation to deactivation is smooth because the field is elastic. This field provides a strong haptic feedback and a negative reinforcement signal to the patient, preventing the approach to the target until the orientation of the bar is recovered.
- *Viscous field.* The purpose of this field, which is proportional to the hand velocity, is to damp oscillations of the hand and stabilize the reaching trajectories. The viscous coefficient that was appropriate for patients was $B = 15 \text{ N/m/sec}$.

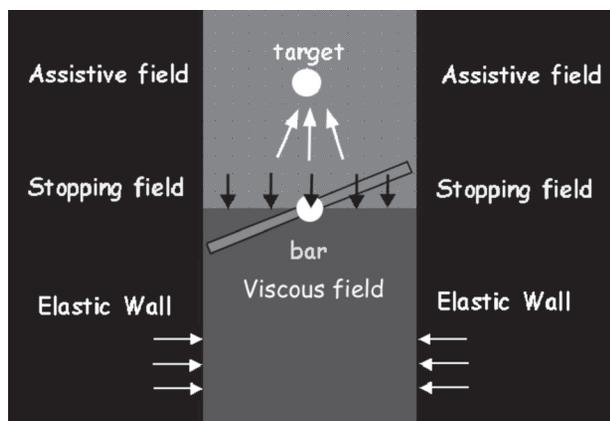


Fig. 2. Combination of force fields implemented by the robot for the designed experimental protocol: (i) an assistive force field directed from the hand to the target; (ii) a stopping field, activated when the orientation error exceeds the threshold ($\pm 4^\circ$); (iii) an elastic wall, for avoiding large lateral deviations from the nominal straight trajectory; and (iv) a viscous field for damping oscillations.

- *Virtual elastic walls.* The purpose of this force field is to avoid large lateral deviations from the nominal trajectory to the target. It has a synergic action to the viscous field, with the purpose of stabilizing the hand while the subject attempts to achieve the target. We chose a rather stiff value: $K_w = 1200 \text{ N/m}$.

The different force fields were simultaneously active and spatially combined in such a way that the haptic virtual environment perceived by subjects was a smooth continuum.

Training sessions were divided into blocks of trials, each of them containing 10 pairs of forward/backward movements. Each session lasted no more than 45 minutes and included a variable number of blocks, as a function of the impairment level. The training cycle consisted of 5 sessions over 2 weeks.

Data analysis

Hand position was evaluated from the measurements of the robot angular rotations, with a precision better than 0.1 mm in the whole workspace, and the corresponding hand velocity was then derived numerically¹. The robot-generated forces could be evaluated directly from the motor currents, taking advantage of the already mentioned very low level of the mechanical impedance of the robot. All these variables were sampled at a rate of 100 Hz. From the recorded data we evaluated simple performance indicators and compared the changes between the first and the last session:

1. the *total number of blocks* of each session, which is proportional to the number of successful reaching movement during the duration of the session (45 min);
2. the level of *assistive force*;
3. the *reaching time* of forward and backward movements, respectively;
4. the *average stopping field*, which is indicative of the number, duration, and entity of the “balance errors” during a reaching movement and thus summarizes the deficit of bimanual co-ordination; also this indicator was evaluated separating forward vs backward movements.

RESULTS

Fig. 3 illustrates the evolution of the motion patterns of one subject from the first to the last session. Initially, the movement profile in the antero-posterior direction is very irregular and decomposed in many sub-movements (top panel) because frequently the bar orientation error exceeds the designated threshold (middle pattern), thus evoking large resistive forces determined by the stopping field (bottom panel), until the subject succeeds to recover the balance between the actions of the 2 arms. The consequence is that the frequency of forward/backward movements is much smaller in the initial than in the final session. At the end of training the motion to the target exhibits rare stop-and-go patterns, the bar orientation error is comprised inside the tolerated interval most of the time and the corresponding resistive force has a very low average value. The overall trajectories in the horizontal plane are shown in Fig. 4.

Table II summarizes the variations between the first and the last session of the previously defined performance indicators. In the first session the most impaired subject (S1: FMA=4) could not complete more than 3 blocks (for a total of 60 forward/backward movements) and this number increased to 6 (for a total of 120 movements) in the last session. In the meantime, the assistance force, necessary for allowing the patients to

¹Time derivatives were computed numerically by using a 4th order Savitzky-Golay smoothing filter, with an equivalent cut-off frequency of 6 Hz.

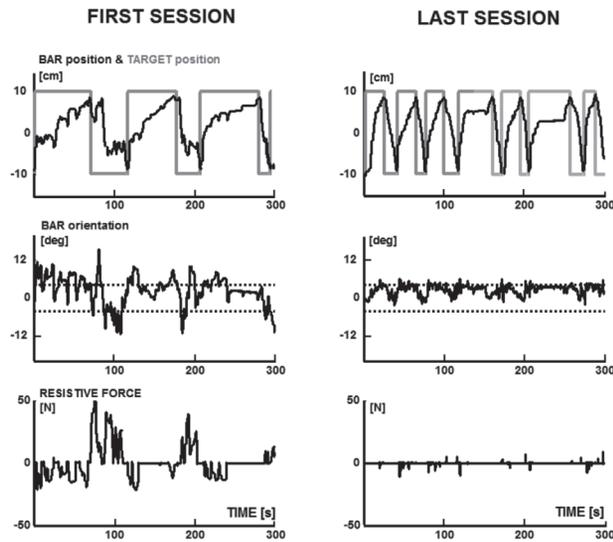


Fig. 3. Evolution of the performance of one subject (S1) from the first to the last session. In the initial session the intensity of the assistive force was 11 N; in the final session it was 3 N. The 2 top graphs display the position of the target (grey trace) and the corresponding position of the bar (black trace) along the antero-posterior direction (positive=forward, negative=backward). The 2 middle graphs show the time course of the bar orientation angle (continuous trace) with respect to the tolerated misalignment ($\pm 4^\circ$; positive=counter-clockwise, negative=clockwise), represented by the 2 dotted lines. The 2 bottom graphs display the resistive forces generated by the stopping force field when the orientation error exceeds the threshold.

carry out the movements was decreased from 25 to 10 N. This pattern (increase of the number of sessions and decrease in the assistive force) was consistent for all the subjects. The reaching time, which was initially over 1 min for the most severe subject, was approximately halved at the end of training for all the subjects, in spite of the large spread of the initial performance that indeed was larger than the spread of the FMA score. On the other hand, the stopping field (the indicator of bilateral coordination) appears to be independent of the initial FMA score, although it consistently decreases with training. Indeed, all the subjects exhibited a consistent adaptive capability, even in the rather short time of the training session, as was confirmed by First/Last *t*-tests of all the indicators. Somewhat surprisingly, the difference between forward and backward movements does

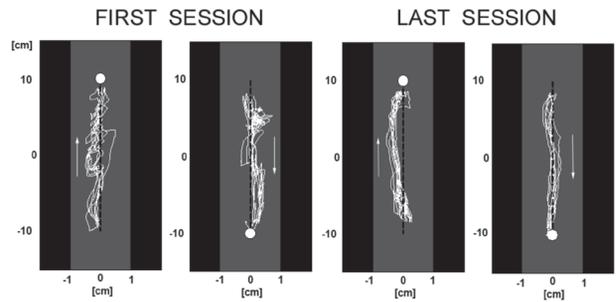


Fig. 4. Trajectories of the centre of the bar (white lines) for forward and backward movements, in the first and the last session, respectively. Positive=forward/rightward; negative=backward/leftward. The circle is the target. The dashed, black line is the nominal trajectory.

not appear to be significant. In another study that involved only movement assistance of the paretic limbs (17), forward movements were systematically slower than backward movements, and this asymmetry is common wisdom in clinical practice. A plausible reason is that the proposed bilateral paradigm, which was designed in order to reinforce balanced bimanual coordination, is also beneficial in reducing the difference between forward and backward movements.

DISCUSSION

In conclusion, this study confirms the promising outcome of bilateral arm training found with the BATRAC (12) and RehaSlide (13) systems. It remains to be seen whether the greater complexity and higher cost of the proposed robot-based bilateral trainer, in comparison with the simpler mechanical systems mentioned above, is justified by a greater clinical potential. No conclusion can be drawn at this point, and controlled clinical trials are necessary as the next step. However, we should emphasize some innovative aspects of the proposed system that exploit the high-performance haptic features of the robot, which are made possible by the direct-drive design. The consequential absence of reduction gears minimizes inertia and friction, and thus allows a truly bi-directional interaction between the robot and the patient: energy flows from the former to the latter or vice versa according to a varying performance and the different phases of a task. Therefore, the robot is not simply a machine that imposes passive movements, as industrial robots would

Table II. Performance indicators of the subjects (S1 to S4)

Subjects	Blocks of trials, n		Assistive force, N		Reaching time, sec				Stopping field, N			
	F	L	F	L	Forward		Backward		Forward		Backward	
S1	3	6	25	20	63.4 (9.9)	28.6 (12.3)	48.9 (3.2)	16.8 (7.4)	5.4 (1.1)	2.0 (1.6)	7.7 (4.2)	1.7 (2.3)
S2	8	10	10	3	18.3 (20.5)	7.7 (3.0)	9.1 (4.8)	6.2 (2.8)	4.9 (2.8)	1.7 (1.9)	5.2 (3.8)	2.0 (2.3)
S3	6	10	8	6	16.6 (7.5)	9.5 (4.6)	10.8 (6.3)	6.7 (2.4)	3.9 (1.4)	2.3 (1.4)	3.1 (1.9)	2.1 (1.7)
S4	7	10	16	4	6.9 (7.6)	4.9 (2.9)	10.4 (10.6)	2.8 (0.6)	5.1 (2.9)	2.3 (1.6)	6.3 (4.0)	2.0 (1.1)
Mean (SD)	6 (2.2)	9 (2.0)	14.7 (7.6)	8.1 (8.0)	26.3 (11.4)	12.7 (5.7)	19.8 (6.2)	8.1 (3.3)	4.8 (2.0)	2.8 (1.6)	5.6 (3.5)	1.7 (1.8)

A "block" of trials consists of 10 "forward" + 10 "backward" movements. The "assistive force" (constant in amplitude after the rise time of 1 sec) is directed from the centre of the bar to the target. The "stopping field" is the average over a reaching movement.

F: first training session; L: last training session; SD: standard deviation.

do, but an agent that helps the patient to relate force and movement, ultimately leading to an improvement in proprioception. The power of the design is also related to the fact that it allows medical personnel without any specific technical know-how to understand the system and define new virtual haptic worlds in a natural way: experimental set-ups and protocols can be conceived at a functional level as combinations of a variety of force fields, modulated by the performance of the patients and sequenced by specific events during the exercises.

Generally speaking, we think that in order to evaluate the impact of rehabilitation technologies one should take a comprehensive view, taking into account that the factors that initiate and maintain cortical reorganization are only scarcely known. In any case, motor rehabilitation is not limited to mechanical/muscular aspects, but is also deeply rooted in motor-cognitive issues, such as motor learning. This is, in our opinion, the mission of exploiting the progressive and unavoidable introduction of haptic robot technologies (18) in the rehabilitation field. Haptics is important because it makes bi-directional interaction between the robot and the patient possible, which makes the causal relationship between effort and error that is important for motor learning available to the brain (19). This will multiply the opportunities to monitor and evaluate in a quantitative way the special type of motor learning paradigm that is recovering motor function in paretic patients. We are confident that the consequent increasing body of knowledge will significantly contribute to an improved understanding of the mechanisms of recovery and the key factors that can enhance it.

ACKNOWLEDGEMENTS

This work was supported by IIT and the FP7 Project Humour of the European Union (EU).

REFERENCES

1. Cramer SC, Bastings EP. Mapping clinically relevant plasticity after stroke. *Neuropharmacology* 2000; 39: 842–851.
2. Carmichael ST. Plasticity of cortical projections after Stroke. *Neuroscientist* 2003; 9: 64–75.
3. Dancause N, Barbay S, Frost SB, Plautz EJ, Chen D, Zoubina EV, et al. Extensive cortical rewiring after brain injury. *J Neurosci* 2005; 25: 10167–10179.
4. Nudo RJ. Mechanisms for recovery of motor function following cortical damage. *Current Opinion in Neurobiology* 2006; 16: 638–644.
5. Jones TA, Chu CJ, Grande LA, Gregory AD. Motor skills training enhances lesion-induced structural plasticity in the motor cortex of adult rats. *J Neurosci* 1999; 19: 10153–10163.
6. Biernaskie J, Corbett D. Enriched rehabilitative training promotes improved forelimb motor function and enhanced dendritic growth after focal ischemic injury. *J Neurosci* 2001; 21: 5272–5280.
7. Kwakkel G, Kollen B, Twisk J. Impact of time on improvement of outcome after stroke. *Stroke* 2006; 37: 2348–2353.
8. Kopp B, Kunkel A, Muhlneckel W, Villringer K, Taub E, Flor H. Plasticity in the motor system related to therapy -induced improvement of movements after stroke. *Neuroreport* 1999; 10: 807-810.
9. Calautti C, Baron JC. Functional neuroimaging studies of motor recovery after stroke in adults: a review. *Stroke* 2003; 34: 1553–1566.
10. Parlow SE, Dewey D. The temporal locus of transfer of training be-

11. Bugar CG, Lum PS, Shor PC, Machiel Van der Loos HF. Development of robots for rehabilitation therapy: the Palo Alto VA/Stanford experience. *J Rehabil Res Dev* 2000; 37: 663–673.
12. Luft AR, McCombe-Waller S, Whittall J, Forrester LW, Macko R, Sorkin JD, et al. Repetitive bilateral arm training and motor cortex activation in chronic stroke. a randomized controlled trial. *JAMA* 2004; 292: 1853–1861.
13. Hesse S, Schmidt H, Werner C, Rybski C, Puzich U, Bardeleben A. A new mechanical arm trainer to intensify the upper limb rehabilitation of severely affected patients after stroke: design, concept and first case series. *Eura Medicophys* 2007; 43: 463–468.
14. Mark VW, Taub E, Morris DM. Neuroplasticity and constraint-induced movement therapy. *Eura Medicophys* 2006; 42: 269–284.
15. Casadio M, Sanguineti V, Morasso PG, Arrichiello V. Braccio di Ferro: a new haptic workstation for neuromotor rehabilitation. *Technol Health Care* 2006; 14: 123–142.
16. Krebs HI, Hogan N, Aisen ML, Volpe BT. Robot aided neuro-rehabilitation. *IEEE Trans Rehab Eng* 1998; 6: 75–87.
17. Casadio M, Giannoni P, Morasso P, Sanguineti V. A proof of concept study for the integration of robot therapy with physiotherapy in the treatment of stroke patient. *Clin Rehabil* 2009; 23: 217–228.
18. Morasso P, Casadio M, Sanguineti V, Squeri V, Vergaro E. Robot therapy: the importance of haptic interaction. *Proceedings IEEE Virtual Rehabilitation, 2007, Venice*, 2007, p. 70–77.
19. Schmidt RA. *Motor control and learning: a behavioral emphasis*. 2nd ed. Champaign, IL: Human Kinetics; 2005.

APPENDIX I. Implementation of the virtual haptic environment.

The virtual haptic environment is implemented by mixing 4 force fields, defined by the following equations:

Assistive field

$$F_a(t) = A \begin{bmatrix} y_t - y_H \\ y_t - y_H \end{bmatrix} T(t) \tag{1}$$

where y_H is the current manipulandum position, y_t is the target position, $R(t)$ is a ramp and hold signal, with a rise time of 1 sec, and A is the amplitude of the assistive field (in N). Therefore the assistive force is directed to the target, whatever the position of the manipulandum position.

Stopping field

$$F_s(t) = \begin{cases} -K_s(y_H - y_{stop}) & \text{if } E > 4 \text{ deg} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

This is a strong elastic field with a stiffness of $K_s = 1200 \text{ N/m}$: y_H is the current position of the hand and y_{stop} is the hand position when the controller detects that the absolute orientation error of the bar E is above a threshold of $\pm 4^\circ$.

Viscous field

$$F_v(t) = \begin{bmatrix} B & 0 \\ 0 & B \end{bmatrix} \begin{bmatrix} \dot{x}_H \\ \dot{y}_H \end{bmatrix} \tag{3}$$

$B = 15 \text{ N/m/s}$ is the viscous coefficient; \dot{x}_H, \dot{y}_H are the time derivatives of the 2 components of the hand position.

Virtual elastic walls

$$F_w(t) = K_w(x_H - x_w) \tag{4}$$

where x_w is the lateral position of the wall and $K_w = 1200 \text{ N/m}$ is the corresponding stiffness.

The robot control mechanism, which implements the virtual haptic environment, iterates the following control loop at the sampling rate of 1000 Hz:

Measure the robot angles $\theta(t)$;

Compute the manipulandum position and speed $x_H(t), y_H(t), \dot{x}_H(t), \dot{y}_H(t)$

Compute the overall force field $F(t) = F_a(t) + F_s(t) + F_v(t) + F_w(t)$;

Compute the robot torques $\tau(t) = J(\theta)^T F(t)$, where $J(\theta)$ is the Jacobian matrix of the robot.

Transform the commanded torques into motor currents.